Proprieta' dei trasformatori

Un trasformatore nella sua forma piu' semplice e' composto da due avvolgimenti, uno chiamato primario, collegato al generatore, l'altro chiamato secondario, collegato all'utilizzatore (carico).
Se l'avvolgimento primario ha un numero Np di spire e l'avvolgimento secondario un numero Ns di spire, una tensione Vp applicata al primario generera' sul secondario una tensione Vs uguale a Vp x Ns / Np.
Se nel primario circola una corrente Ip allora la corrente nel secondario Is sara' uguale a Ip x Np / Ns.
Si puo' indicare Is x Ns = Ip x Np.
La potenza sul primario e' uguale a quella su secondario (supponendo un trasformatore senza perdite).
L'impedenza sul primario e' Zp = Vp / Ip, quella sul secondario e' Zs = Vs / Is
Sa queste relazioni si ottiene Zp = Zs x (Np / Ns $)^2$
Il rapporto Np / Ns che sembra regolare il funzionamento del trasformatore e' chiamato *rapporto di trasformazione*

Se il primario ha 10 volte il numero di spire del secondario (rapporto di trasformazione = Np / Ns = 10), la tensione sul secondario sara' 10 volte piu' piccola di quella al primario, la corrente circolante nel secondario sara' 10 volte piu' grande di quella circolante nel primario, l'impedenza sul primario sara' 100 volte piu' grande di quella sul secondario

Induttanza

Sembrerebbe che il solo rapporto di trasformazione sia il parametro da considerare, nulla ci impedirebbe di pensare a un trasformatore costituito da 5 spire sul primario e una sola spira sul secondario.
Supponiamo che il secondario non sia collegato ad alcun carico, cioe' sia a circuito aperto.
Se noi applichiamo una tensione alternata al primario, la corrente sara' limitata dalla impedenza costituita dall'autoinduzione o induttanza del primario: Ip = Vp / (L x omega), dove omega = 2 x pi x f (pi = 3,14..; f = frequenza misurata in Hz).
Questa corrente, da considerare una perdita, poiche' il trasformatore non produce alcuna potenza (funzionamento a vuoto), dovra' essere la piu' piccola possibile.
Per una corrente a frequenza molto bassa, il valore di omega sara' molto piccolo, a 20 Hz il denominatore di Ip sara' 125,6 (2 x 3,14 * 20), il valore di L dovra' essere molto grande per constituire un carico sufficiente alle basse frequenze.
L'induttanza dipende dal numero di spire dell'avvolgimento e inoltre dalla permeabilita' del nucleo magnetico su cui e' avvolto.
L'induttanza e' quindi un valore molto variabile, perche' mentre il numero di spire e' fisso non lo e' la permeabilita' del nucleo.
La necessita' di avere un primario con un grande numero di spire aumenta la lunghezza del filo necessario e la sua resistenza ohmica. Quest'ultima dovra' essere la piu' piccola possibile.

Il trasformatore per frequenze audio

A differenza di un trasformatore di alimentazione che funziona a tensione costante sul primario e a frequenza fissa, il trasformatore per frequenze audio deve lavorare a tensione e frequenza variabile. Inoltre il primario e' percorso da una corrente continua di valore medio piu' o meno costante secondo il tipo di funzionamento delle valvole (classe A oppure B, push-pull oppure single-ended).
I trasformatori per frequenze audio sono progettati in modo che le correnti continue che li attraversano siano 'neutre' al loro funzionamento.
La corrente continua e' dovuta al punto di lavoro delle valvole. Nei montaggi push-pull, le correnti nelle due valvole sono uguali e circolanti in senso inverso. Il loro effetto sulla magnetizzazione del nucleo si annulla.

Transformer Math

There are lots of books that cover this subject but it is not easy to pull the information together into a friendly format.
Below is an example calculation of the electrical characteristics, number of turns and air gap for an output transformer suitable for use with a 300B.
I should point out that the choice of core type and material is up to you. I have very limited data on cores so please don't email asking for data.
However I will be pleased to help with the math if you have the data, also to receive helpful comments or corrections if you find any error or know a better technique / formula.

After the example, the theoretical basis of the equations is given.

SUMMARY OF FORMULAE (using CGS units):

Reactance of an inductor (Ohms):

X = 2.ð.f.L or L = X/2.ð.f

Equivalent permeability of air gapped magnetic circuit μ_e:

$\mu_e = \mu / (1+\mu(l_g/MPL))$

µ is the material permeability relative to μ_0 .

DC Flux (Gauss):

$B = 1.257 \times \mu_e . N.I/MPL$ or $N = B.MPL / 1.257 \times \mu_e .I$

Inductance (Henries):

$L= 1.257 \times \mu_e.N^2.A \times 10^{-8}/MPL$

Peak AC Flux (Gauss)

$B_{AC} = Vr.m.s \times 10^8/4.44.N.f.A$

Transformation Ratios:

$Vp/Vs = Np/Ns = (Zp/Zs)^{1/2} = (Lp/Ls)^{1/2}$

CGS Units:
I Amperes f Hz A sq cm MPL cm l_g cm

APPLICATION OF FORMULAE TO AUDIO TRANSFORMER DESIGN

Flux density is the key consideration for the design of an audio transformer. We do not want the core to saturate at-all. In fact, we want to stay within the sensibly linear region of the B-H curve.

The transformer equation shows that the AC flux density increases with decreasing frequency so we need to consider the lowest frequency of interest.
If the transformer is to handle DC current, we need to consider not only the sum of the DC flux and the peak AC flux but also, the effect of the DC current on the primary inductance of the transformer.

Let $I_{DC} = 0.08A$

Now we have a dilemma, we have to chose core material and dimensions.
This is where either experience or a lot of iteration comes in.
For the purposes of this example let's use a core having the following parameters:

MPL = 26cm

$\mu = 4000$ for silicon steel C core with MPL = 26

$A = 11.6cm^2$ (Note, this figure takes into account the stacking efficiency of the core.)

Now we come to the next dilemma, we know that a gap is likely to be required because the DC current is comparable to the peak AC current in this application. In practice, it is a good idea to use a gap large enough that variations in DC current will not affect the primary inductance greatly. Experience suggests that a *total* gap length of 22mils (0.056cm) might be suitable.

$l_g = 0.056cm$

PROCEDURE.

1/ Determine a target value for the primary inductance, Lp. We know that the plate resistance of a 300B is 700 . The primary inductance controls the LF performance so let us make 10Hz the lowest frequency of interest.

A good 'rule of thumb' (for a SE triode) is to make the reactance at the lowest frequency of interest equal to 5 times the plate resistance. For an inductor,

L = X/2.ð.f

700 x 5 = 3500

10Hz is the lowest frequency of interest:

L = 3500/2. ð.10 = 56H

2/ Calculate the effective permeability of the core with an air gap of 0.056cm:

Using $\mu_e = \mu / (1+\mu(l_g/MPL))$;

$\mu = 4000 / (1+4000 \times (0.056/26)) = 416$

3/ Calculate number of turns permissible with the given DC current:

Using $N = B.MPL / 1.257 \times \mu_e .I$;

For the given type of core, a good limit for the total flux density is 16,000 Gauss. At this level, the transformer will remain linear. We know that Idc = 0.08A. For a 300B, we can also see that the peak AC current will approach but not exceed the DC current and thus we can set a limit for the DC flux density as one-half of the total flux density, 8000 Gauss.

N = 8000 x 26 / 1.257 x 416 x 0.08

N = 4972, say 5000 turns.

Now we have another dilemma, how many turns are practical? For this we need to evaluate the core window against the chosen wire size, insulation and secondary turns. In this case, I know (again from experience) that 5000 turns will fit comfortably.

4/ And so we can calculate the primary inductance:

Using; $L= 1.257 \times \mu_e.N^2.A \times 10^{-8}/MPL$;

$L = 1.257 \times 416 \times 5000^2 \times 11.6 \times 10^{-8} / 26$

L = 58H This is just above the desired target of 56H.

(Yes, I did 'cheat' by doing all the necessary iteration before setting out this example. This is where the work lies……)

5/ Calculate the AC flux at 10Hz:

Using; B_{AC} = Vr.m.s x 10^8/4.44.N.f.A;

Also; Vp/Vs = Np/Ns = $(Zp/Zs)^{1/2}$ = $(Lp/Ls)^{1/2}$

Another good ‘rule of thumb’ is to make the primary impedance somewhat greater than 5 times the plate resistance*, say 3800 . This will give efficient power transfer while allowing for speaker impedance dips.

We want a primary impedance of 3800 and let’s say that the secondary is to be 8 , the turns ratio will be:

Np /Np = $(Zp/Zs)^{1/2}$ = $(3800/8)^{1/2}$ = 22

We know that the turns ratio is also the voltage transformation ratio.

For a power of 10W into 8 : Vrms = $(10 \times 8)^{1/2}$ = 8.9Vrms.

Thus for 10W, the voltage across the primary will be 22 x 8 = 196Vrms

B_{AC} = 196 x 10^8/4.44 x 5000 x 10 x 11.6 = 7600Gauss

6/ Check combined DC and peak AC flux:

The AC and DC flux densities sum to 15600Gauss. This is the maximum value at 10Hz at 10W.

Silicon steel C cores saturate at more than 16000Gauss so this is satisfactory.

* You should always confirm this by plotting the load-line and then make an allowance for load reactance which will cause the load-line to become elliptical.

BASIC THEORY OF ELECTROMAGNETIC INDUCTANCE.

Analogy between the magnetic circuit and the electric circuit:

Analogy between the magnetic circuit and the electric circuit:

Electric Circuit		Magnetic Circuit	
Quantity	Unit	Quantity	Unit
E.m.f Current Current density Resistance	Volt Ampere Ampere/m^2 Ohm	m.m.f Magnetic field strength H Magnetic flux F Magnetic flux density B Reluctance S	Ampere Oersteads Maxwells Gauss Ampere/Maxwell
Current = e.m.f./ resistance		Flux = m.m.f./reluctance	

S.I. definition of electrical and magnetic units and CGS conversions:

Unit	Symbol	Definition	To convert to CGS	Multiply by
Henry (H)	L	The *inductance* of a closed circuit in which an e.m.f. of 1V is produced when the electric current varies at a rate of 1A/s		
Weber (Wb)	F	The *magnetic flux* which, when linking a circuit of one turn, produces in it an e.m.f. of 1V when it is reduced to zero at a uniform rate in 1s	Maxwells & lines	1×10^8
Ampere/metre	H	Ampere-Turns per metre (Ampere-Turns per cm)	Oersteads	0.01257 (1.257)
Tesla (T)	B	The *magnetic flux density* equal to 1 Wb/m^2	Gauss & Lines/cm^2	10^4
Henry/metre	m_0	Permeability of free space $4px10^{-7}$	Gauss/Oerstead	795774.72 (= unity)

MAGNETIC FIELD STRENGTH.

H is the *magnetic field strength* due to a current flowing in a coil.

H = *magneto motive force* (mmf) per per unit length of the magnetic circuit.

The length of the magnetic circuit is denoted by MPL

The mmf is the force caused by a current I flowing through N turns. In a coil it is the *total* current linked with the magnetic circuit.

The unit for H is the Oerstead which is equal to 1.257 ampere-turns per cm.

Thus H = 1.257xN.I/MPL (Oersteads)

DC FLUX DENSITY.

Consider a point C on magnetic field of radius r about a conductor A situated in a vacuum:

B is the flux density at point C

DEFINITION: C r
m0 C

thus;

$B=\mu_0.H$

Substituting for H from above we have:

$B = \mu_0 \times 1.257xN.I/MPL$ (Gauss)

NOTE, In CGS units, $\mu_0 = 1$ Gauss/Oerstead

SELF INDUCTANCE.

1 1
1 10^8

Ö is the *total magnetic flux* produced by a current flowing in a coil.

The unit for Ö is the Maxwell or line.

Ö = B.A where;

B is the *flux density*. (Gauss)

A is the core cross sectional area. (cm^2)

Flux linkage is the linkage between the number of lines of flux .

Thus the *total* flux-linkage = Ö x N

Since flux is proportional to current and total flux linkage is proportional to flux, then it follows that flux-linkage is proportional to current thus;

Flux-linkage I.

Introducing a constant, k we have;

Flux-linkage = k.I

This constant, k is the *self inductance* of the circuit (electrical and magnetic) and is given the symbol L. Note that by definition, 1H results when 1A produces 10^8 lines-turn thus we must *divide* the result by 10^8; Making inductance, k (L) the subject we have;

L=Flux-linkage/I x 10^8

From above, flux-linkage = Ö x N, more usually written N.F, we have the result;

L= N.Ö/ I x 10^8

From above, we have Ö = B.A, B=μ_0.H and H = 1.257 x N.I/MPL

Thus,

Ö = 1.257 x μ_0.N.I.A/MPL, substituting into the expression for L we get;

L= 1.257 x μ_0.N^2.A x 10^{-8}/MPL (H)

Observe that L N^2 and so we can further state:

$Lp/Ls = (Np/Ns)^2$

A further essential result can be derived at this point, the relationship between turns ratio and primary to secondary impedance.

Neglecting losses, the primary power will be transferred to the secondary. Ohm's law gives,

$P = V^2 / R$. In this case, R will be impedance, thus so:

$Pp = Vp^2 / Zp = Ps = Vs^2 / Zs$

Now, the voltage in the primary and secondary windings is *directly* proportional to the number of turns and so we can replace V with N which yields the important result:

$Np^2 / Zp = Ns^2 / Zs$ Cross multiplying we get:

$(Np /Np)^2 = Zp / Zs$ which, from above is also equal to Lp/Ls thus:

$Vp/Vs = Np/Ns = (Zp/Zs)^{1/2} = (Lp/Ls)^{1/2}$

PERMEABILITY:

Since we are considering iron (or other magnetic material) cored transformers, we need to modify the permeability from that of free space to that of the core. The permeability of magnetic materials is usually specified as *relative permeability* to that of free space. The permeability is also a function of magnetic path length. Permeability data for core materials is sometimes presented as a graph of permeability vs MPL.

Thus $\mu = \mu_0.\mu_r$

Hereafter, μ will represent the product, $\mu_0.\mu_r$.

INDUCTOR WITH AN AIR GAP:

Consider an inductor having an air gap where subscript g indicates the gap and subscript m indicates the core:

Total reluctance = $l_g/\mu_g.a_g + MPL/\mu_m.a_m$

Now we can introduce *equivalent permeability* for the complete circuit, μ_e

The total reluctance will be equal to $l_t/\mu_e.a$

We can now equate these two formulae for total reluctance. Noting that the area of the gap is equal to the area of the core and so multiplying through by a;

$l_t/\mu_e = l_g/\mu_g + MPL/\mu_m$

Note, μ_m is the product, $\mu_0.\mu_r$ which we will refer to as μ

Cross multiply to make μ_e the subject and we have;

$\mu_e = l_t/(l_g/\mu_g + MPL/\mu)$

$l_t = l_g + MPL$ =>

$\mu_e = l_g + MPL/(l_g/\mu_g + MPL/\mu)$

For any likely design, $l_g <<< MPL$ we can multiply both sides by $\mu/MPL/\mu/MPL$;

$\mu_e = \mu / (1+\mu(l_g/MPL))$

This is the classic equation for equivalent permeability of a core – air gap magnetic circuit. It is used to modify the equations for DC flux density and thus also, inductance.

AC FLUX DENSITY;

To calculate the flux density due to AC current, we need to use Faraday's law of electromagnetic induction:

DEFINITION:

Consider a single sinusoidal magnetic flux wave of peak amplitude Ö and frequency f. The flux will change from +Ö to -Ö in ½.f seconds thus;

Average rate of change of flux = 2Ö ¸ 1/2f = 4fÖ Webers/second.

By definition, 1 weber/second = 1 volt and noting that 1Wb 10^8 lines, we have; Average e.m.f. induced per turn = 4fÖ volts and the r.m.s. value is 1.11 times the average value and thus;

Vr.m.s. / turn = 4.44fÖ/10^8 volts and for N turns;

Vr.m.s = 4.44NfÖ/10^8 volts. Or:

N = Vr.m.s x10^8 /4.44.f.Ö This is the classic transformer equation (which is used to calculate the number of turns required for the primary of a power transformer for a desired maximum flux).

Re-writing the transformer equation to make Ö the subject and knowing that Ö= flux density times area we have the peak AC flux density;

B_{AC} = Vr.m.s x 10^8/4.44.N.f.A (Gauss)

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Langford-Smith Radiotron Designer’s Handbook
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Transformer Frequency Response

Here's a proposal for modelling the frequency effects of a transformer.
Comments regarding appropriateness or accuracy are welcome.

Feedback and Compensation FAQ:

Transformer Frequency Response Model:

Transformers are more complex than RC networks from a frequency response view. The following formulas allow you to predict the frequency response of your circuit with transformer coupling.

Definitions:
Lp = Primary inductance
LL = Leakage inductance
Cp = Primary winding capacitance
Cs = Secondary winding capacitance
n = Turns Ratio
f = Low Frequency pole
fr = High Frequency resonant point
rp = Plate resistance (also add primary winding resistance)
Rl = Load resistance (also add secondary winding resistance)
Q = Circuit Q of the high frequency pole pair

Low Frequencies:
Pole:
Unterminated transformer case:
  f = rp/(2*pi*Lp)
Terminated transformer case:
  The resistance in the above formula is the parallel of the primary side resistance (rp) in parallel with the load resistance (rl) referred to the primary (making it rl*n*n) so...
  f = Rl*rp*n*n/(2*pi*Lp*(Rp + (n*n*rl)))

Zero: There is a zero at DC

High Frequencies:

Poles:

There is a set of "complex" hi frequency poles. This leads to peaking at high frequencies as has been discussed in this newsgroup. The formulas given allow you include the effect of these on your feedback circuit. Since the pole-pair is "complex" I have to introduce the concept of "Q". Higher Q leads to more peaking, lower Q to less peaking. The following table provides an insight into how much high frequency peaking to expect:

Q	Peaking (dB)	-3dB point W.R.T. "resonant" freq.
2	6 dB	1.5
1.41	3 dB	1.4
1	1 dB	1.25
.8	0.3 dB	1.1
.7	0 dB	1
.6	no peaking	0.8
.5	no peaking	0.65

Resonant Frequency:

fr = 1/(2*pi*sqrt(LL*(Cp+(n*n*Cs)))

Circuit Q:

Q = (2*pi*fr*LL)/(rp paralleled with (n*n*Rl))

Example:

A 1:1 transformer couples a 6SN7 plate into the grid of a class AB1 triode. The grid resistance is assumed high. (rp = 7.7k)

Transformer characteristics

Lp = 25 Henry

LL = 0.05 Henry

Cp = 100 pF

Cs = 100 pF

Winding resistance assumed negligible.

Use "unterminated" cases:

Low Frequency pole is at 7700/(2*pi*25) = 49 Hz.

Hi Frequency Resonance is at 1/(2*pi*sqrt(0.05*200pf) = 50.3 kHz

Q is (2*pi*50300*0.05)/7700 = 2.05

From the table, the -3 dB point is 1.5*50300 = 75.5 kHz.

There will be a 6 dB peak in the response at about 50kHz.

Transformer Low Frequency

Of course there is the Lenz law which " works " by providing a magnetic field opposed to the disappearance of the current. This natural effect works well as much as the primary inductance of the output transformer is enough to store the energy; and there is the fundamental importance of the design of a generously sized output transformer if we wish to have a good reproduction of the low frequencies.

The energy stored in the primary is:

QJoule = 1/2 * LHenry * I2Amp

For example in a 300B stage, the idle current is generally 75mA when a good transformer has a 20 Henrys primary inductance.
In this case :

Q = 1/2 * 20 * 0,075 * 0,075 = 0,056 J

1 J = 1 W /sec

Let us calculate the distribution of this power according to the frequency we have to reproduce:
At 20Hz the time necessary for the inductance to give back energy is 0,025 seconds.
During this time the inductance is able to give back a power of :

PWatt = 0,056J * 1 / 0,025 = 2,24 W 20Hz

It is not much and explains the fall of the power in the bass extremities.
At 100Hz the time of return is 0,005 seconds. The corresponding available power is:

PWatt = 0,056J * 1 / 0,005 = 11,2 W 100Hz

This power is widely appropriate and the respone at 100Hz is practically perfect .
The same calculation for the frequency 1000Hz reveals now why Mono Triodes amplifiers have a musical power reserve so amazing. At 1000Hz the time of restitution is 0,0005 seconds that gives a disposable power of:

PWatt = 0,056J * 1 / 0,0005 = 112 W 1000Hz

Here we ought to compare with what happens in a push pull amplifier, it is easy because of their principle .
The electricity stocked in the primary of the output transformer is always non-existent, and therefore the profitable phenomenon described above is not present.

In the future, I will comment about the reserves written at the beginning of this technical note. My assumptions are not yet confirmed by well-know scientific studies and I am waiting for the confirmation of a mathematician .